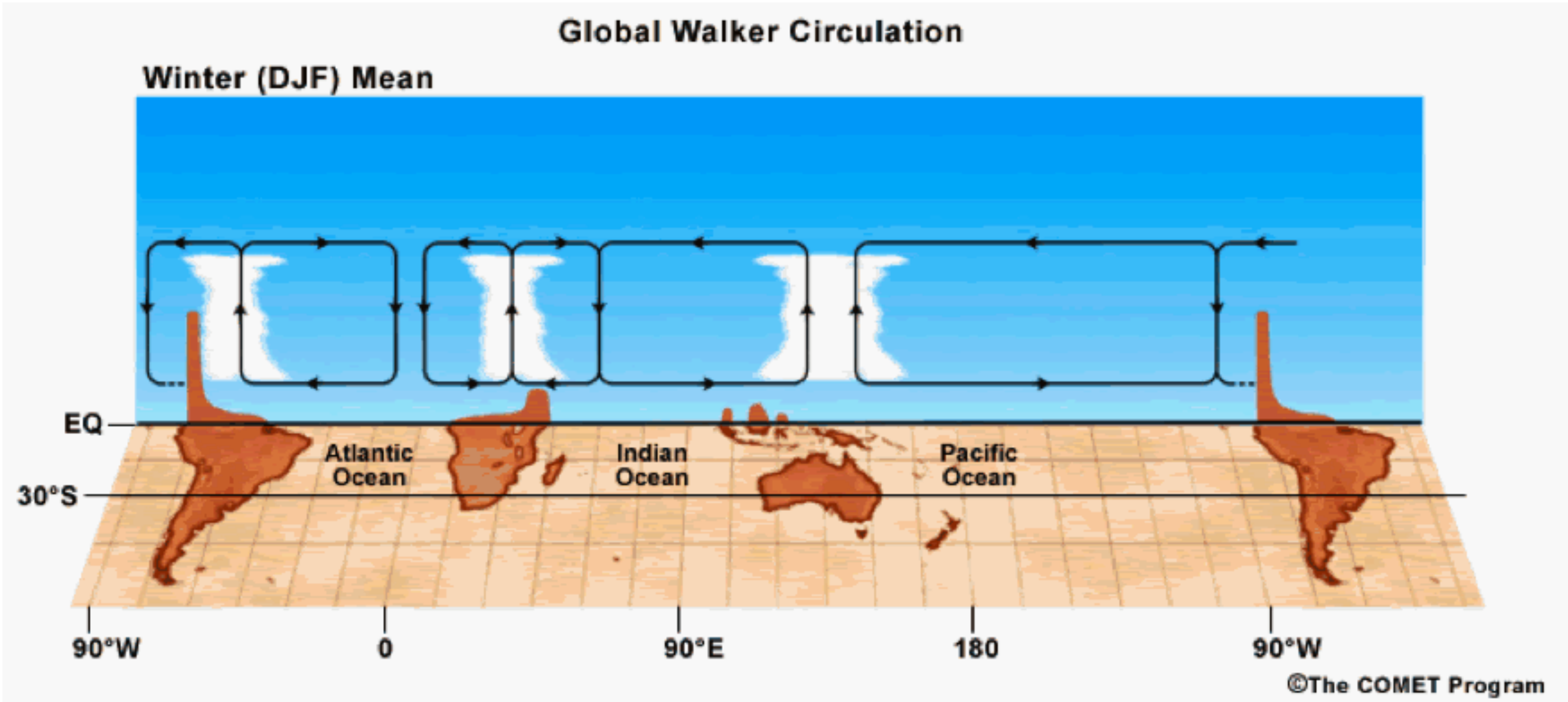
- Description of the Walker circulation
- Weak temperature gradient (WTG) approximation

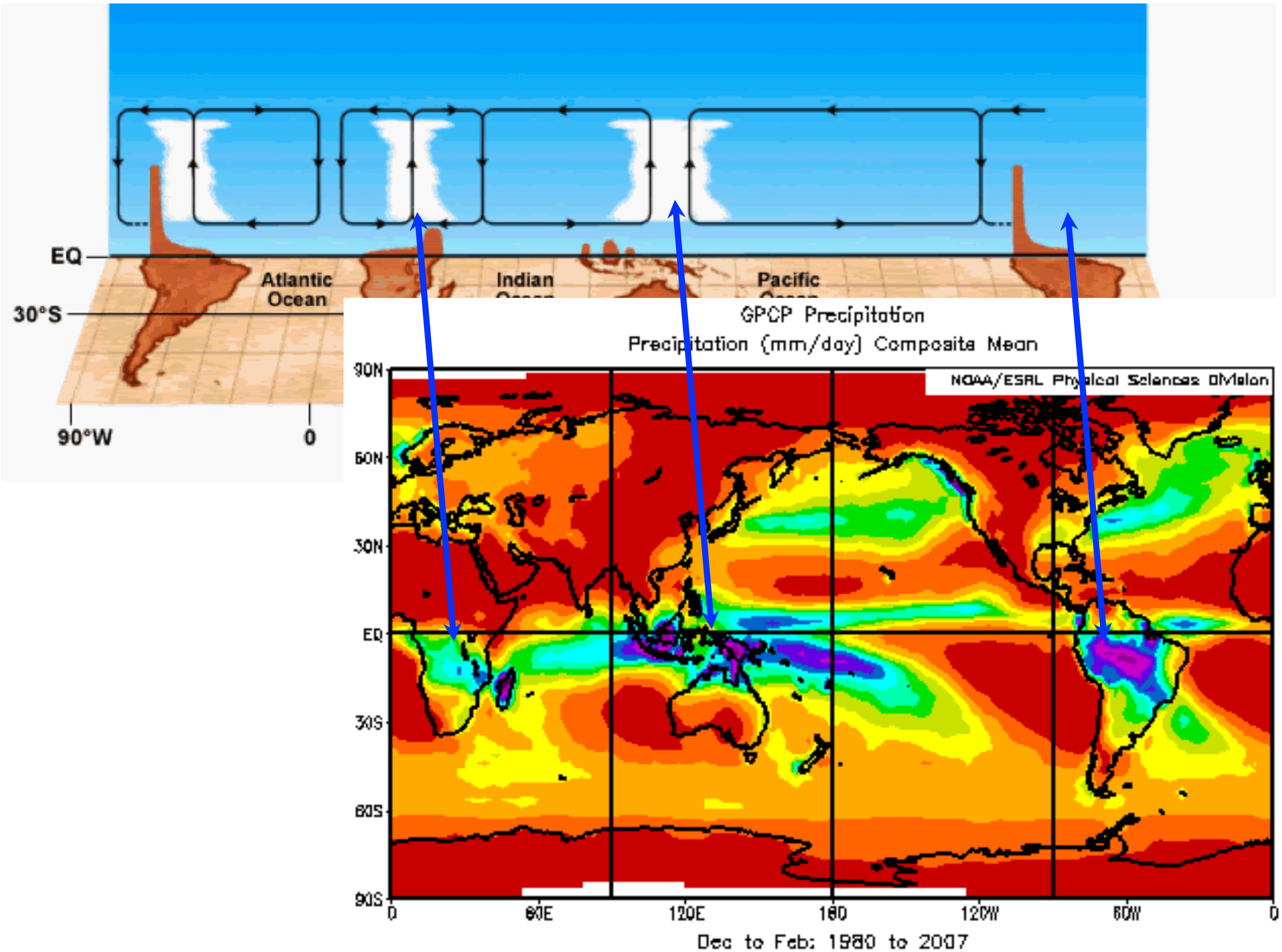
Goal: Describe the Walker circulation and formulate the weak temperature gradient (WTG) approximation

Overview of the Walker circulation

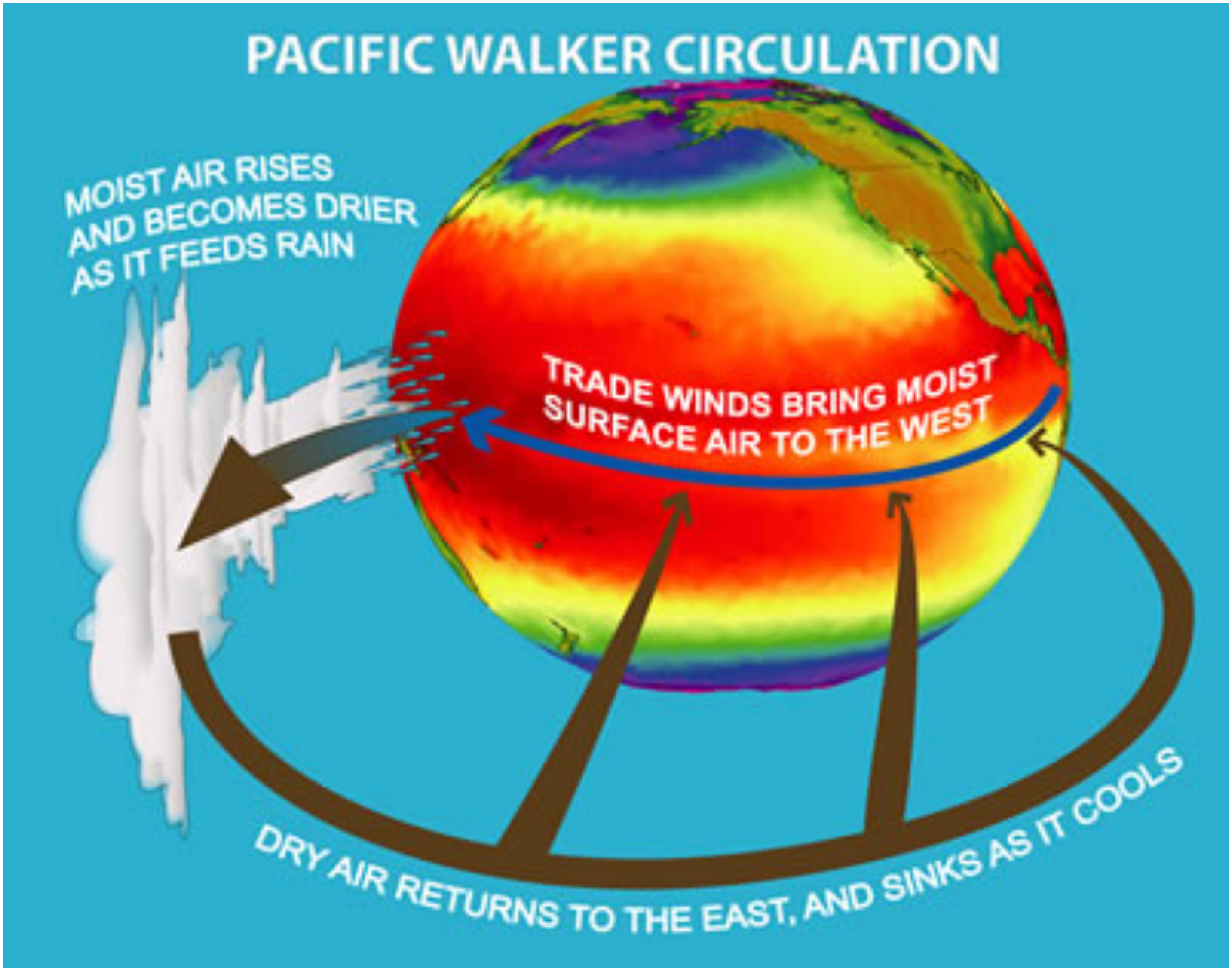


- The Walker circulation is a 2D circulation along the equator in the longitude-height plane.
- This circulation consists of cells comprising rising motions in major centers of equatorial convection (the Amazon, Africa, western Pacific) and sinking motions over adjacent ocean basins.

DJF Precipitation (mm day^{-1})

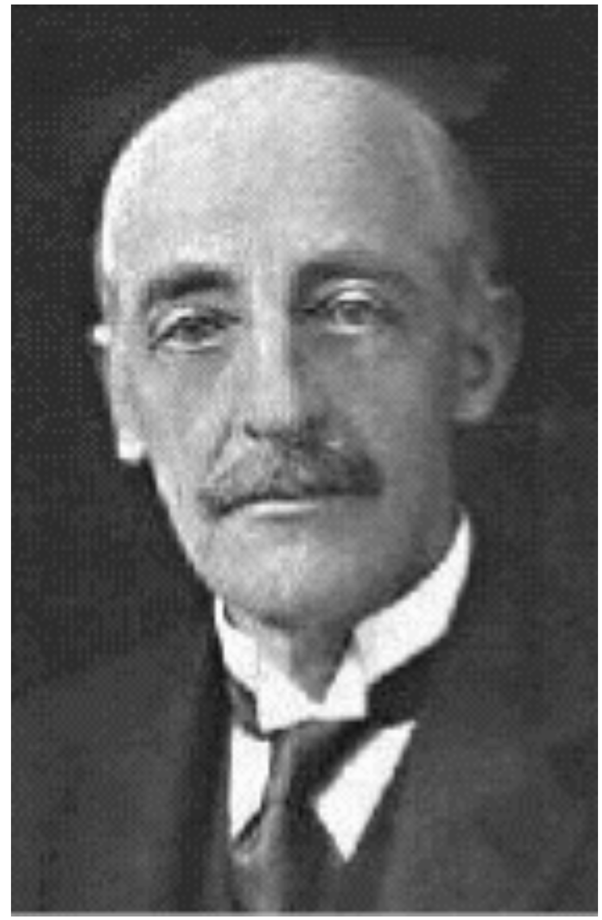


The Pacific Walker cell



Sir Gilbert Walker (1868-1958)

- Jacob Bjerknes coined the term Walker circulation in 1969, in honor of the British physicist and statistician who was director-general of observatories in India from 1904-1924
- While in India, Walker focused on statistical relationships among temperature and rainfall, first identifying the Southern Oscillation.
- Walker authored or co-authored the awesomely-named sequence of papers: *World Weather I - VI*



For an on-line presentation about Walker see:

http://www.walker-institute.ac.uk/media/GILBERT_WALKER.pdf

Walker circulation in the NCAR Reanalysis

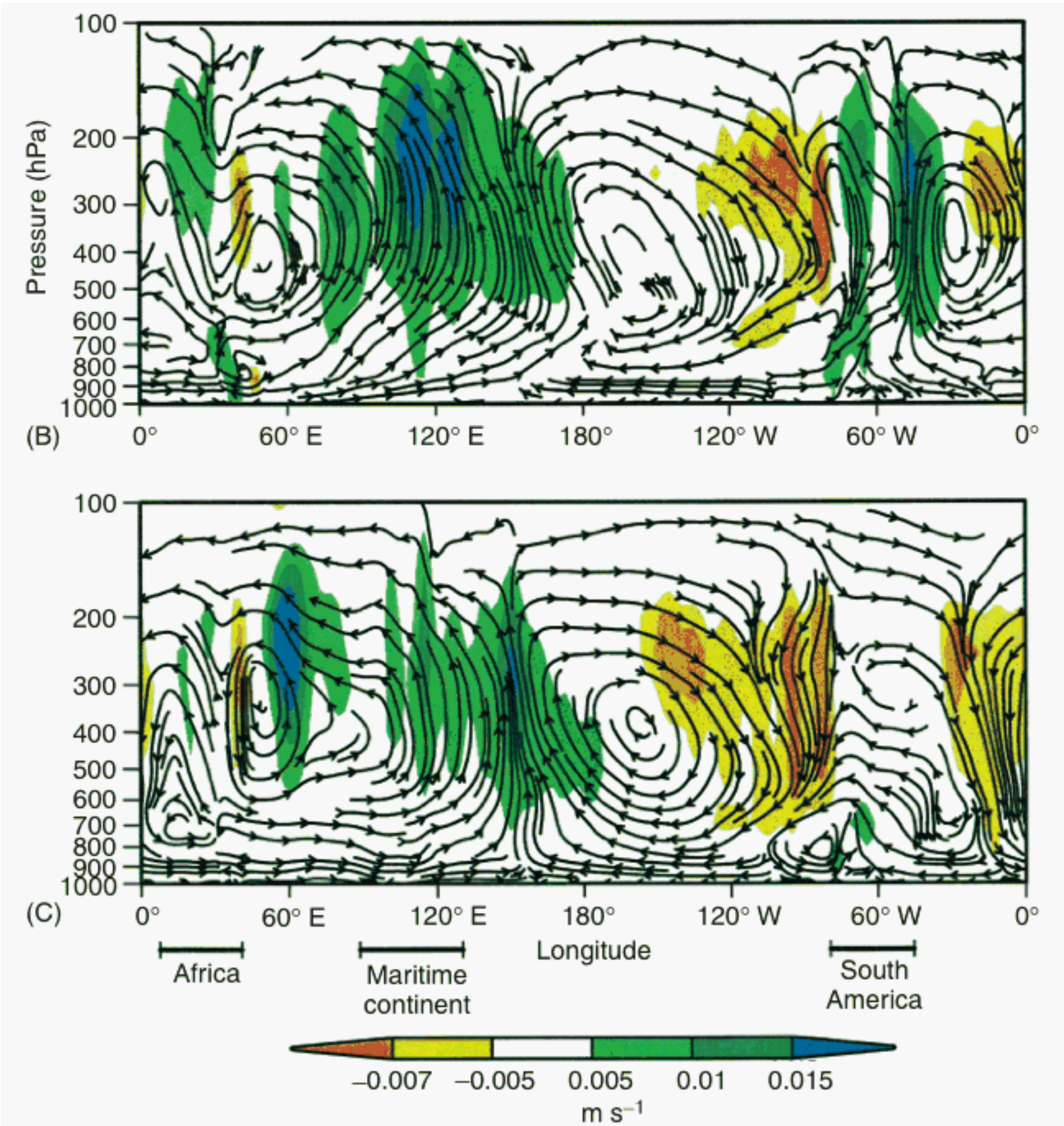


Figure from Lau and Yang, 2002

- Shown here are streamlines constructed from the divergent component of the equatorial wind field for January (top) and July (bottom).
- Areas of strong ascent/ descent are shaded
- Some seasonality is evident, e.g., the strong South America/Atlantic cell in January is absent in July.

A theoretical framework for large-scale tropical circulation*

One of the consequences of the smallness of the Coriolis parameter near the equator is that horizontal temperature gradients in the tropical atmosphere are small. In turn, this condition provides constraints on the large-scale dynamics and their relationship to diabatic processes.

We can take advantage of this to build a balanced dynamical framework for the tropical circulation (or at least its divergent part). This framework is known as the weak temperature gradient (WTG) approximation.

Recall from Helmholtz's theorem, or the fundamental theorem of vector calculus, that we can decompose an arbitrary vector $\vec{F}$ as:

$$\vec{F} = -\vec{\nabla}\Phi + \vec{\nabla} \times \vec{A}$$

where Φ and $\vec{A}$ are scalar and vector potentials, respectively.

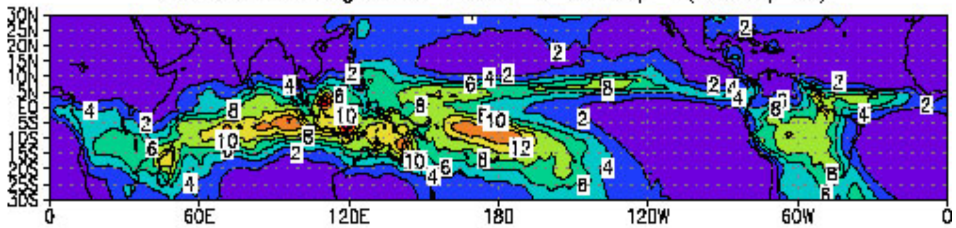
In particular, we can use this result to write the velocity field in terms of rotational (nondivergent) and divergent (irrotational) components:

$$\vec{v} = \vec{v}_{rot} + \vec{v}_{div}$$

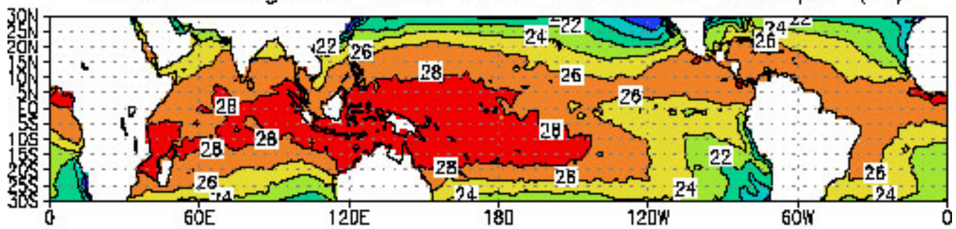
*This material adapted from Professor Adam Sobel's lecture notes at the 2005 GCC summer school [see: http://www.atmosp.physics.utoronto.ca/MAM/summeragenda05.html]

WTG Justification

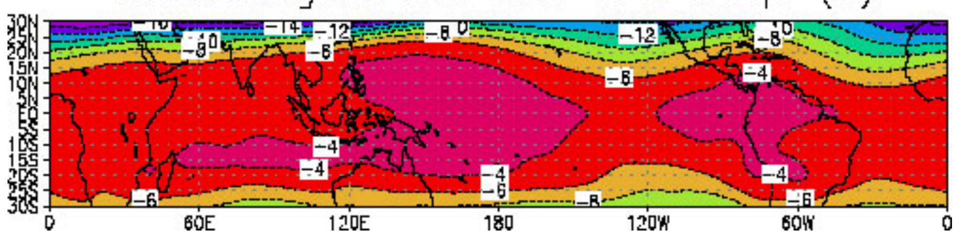
Climatological Jan. Precip (mm/d)



Climatological Jan. Sea Surface Temp. (C)



Climatological Jan. 500 hPa Temp. (C)



Thermodynamic equations

Start from primitive equations for dry static energy (energy conservation) $s = c_p T + gz$ and specific humidity q (conservation of water mass):

Note: Quantities on the lhs should be regarded as representing large- (or grid-scale) values

$$\frac{\partial s}{\partial t} + \mathbf{u} \cdot \nabla s + \omega \frac{\partial s}{\partial p} = Q_c + Q_R \qquad (1)$$

$$\frac{\partial q}{\partial t} + \mathbf{u} \cdot \nabla q + \omega \frac{\partial q}{\partial p} = Q_q \qquad (2)$$

In (1), Q_c is the convective heating and Q_R is the radiative heating, and in (2), Q_c is the convective drying. Note that integrating the convective heating and drying over the depth of the troposphere, i.e., from the surface to the tropopause, yields:

$$\int_{p_s}^{p_t} Q_c dp = P + \overline{\omega' s'}|_{p_s} = P + H$$

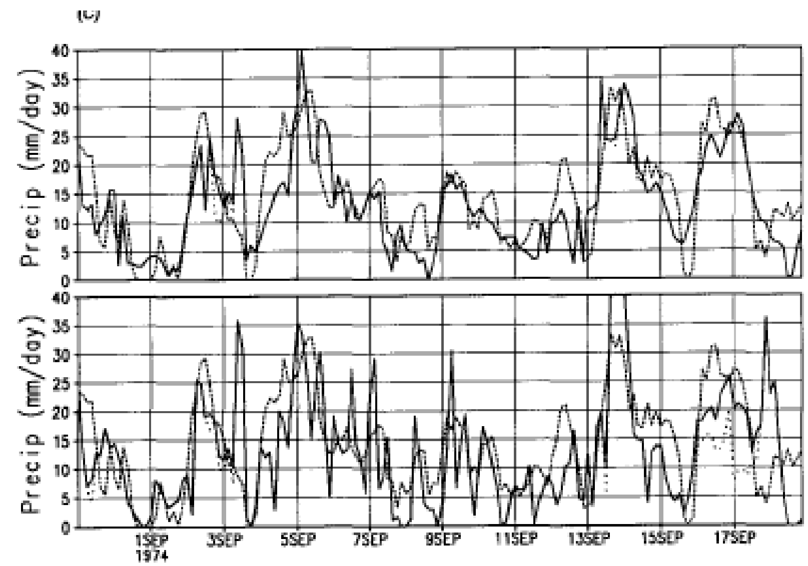
$$\int_{p_s}^{p_t} Q_q dp = -P + \overline{\omega' q'}|_{p_s} = -P + E$$

Note: Sensible and latent heat fluxes (H and E) are related to surface-layer turbulent transport of dry static energy and moisture, respectively.

Dominant thermodynamic balance for tropical precipitating conditions

$$\frac{\partial s}{\partial t} = -\mathbf{u}\cdot\nabla s - \omega\frac{\partial s}{\partial p} + Q_c + Q_R$$

$$\frac{\partial q}{\partial t} = -\mathbf{u}\cdot\nabla q - \omega\frac{\partial q}{\partial p} + Q_q$$



The dominant balance for tropical precipitating conditions is:

$$\omega\frac{\partial s}{\partial p} = Q_c + Q_R$$

Typically, $Q_c >> Q_R$, and $\partial s / \partial p \sim \text{constant}$, so $Q_c \propto \omega$. Since $P \propto \int_{p_s}^{p_t} Q_c dp$, specifying the vertical velocity profile is effectively determines the rate of precipitation. (One can consider this the conventional view in which vertical velocity is *a priori* known, or at least determined by the momentum equations.)

WTG equation

On the other hand, in the deep tropics, we could think of ω as if it is *completely determined by* the diabatic heating. In this sense, the equation for dry static energy reduces to the dominant balance:

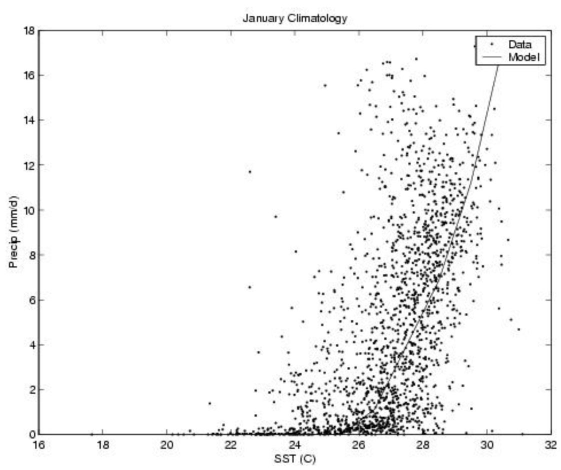
$$\omega \frac{\partial s}{\partial p} = Q_c + Q_R$$

This WTG equation is diagnostic for ω, rather than prognostic for dry static energy (or temperature). Thus, we cannot predict temperature itself, but that may be fine for many purposes because it does not vary much anyway in the (deep) tropics. On the other hand, we can immediately relate the divergent circulation [consider mass continuity] to the diabatic heating!

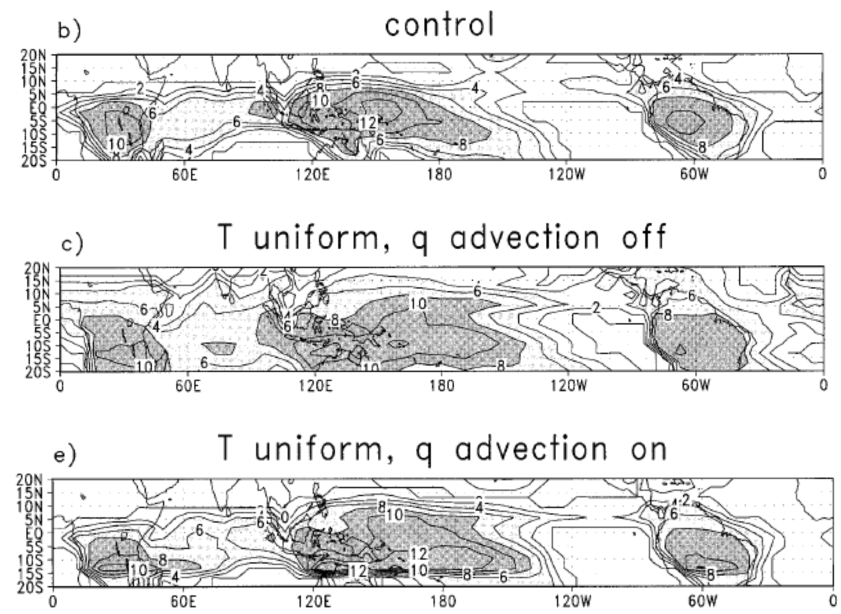
We can solve a Poisson's equation (using ω) to obtain WTG-consistent horizontal wind components.

Example WTG simulations

(from Sobel and Bretherton 2000)



Comparing the observed SST-P relationship to that computed using a radiative-convective model (RCM) with a fixed vertical temperature profile and no moisture advection.



Comparing precipitation as simulated in the Neelin and Zeng (2000) Quasi-equilibrium Tropical Circulation Model (QTCM) to WTG RCMs with identical physics at each grid point.